Losses in Planar Waveguides

Maik Holzhey and Richard Schulz

***Abstract*—A comprehensive study to calculate losses in planar waveguide structures is presented theoretically with approximate formulas and numerical simulation, respectively. Results are critically reviewed by comparison of the two approaches. Approximate formulas to calculate line impedance, ohmic and dielectric attenuation are given. CST MICROWAVE STUDIO®[1] is used to perform the numerical calculations and validate the formulas' approximate results. Furthermore, a summary of relevant research, referring to losses in planar waveguides, is provided.**

I. INTRODUCTION

In the 1960th and 1970th has been a strong urge to determine important electrical properties of a planar waveguide, consisting of two metal planes separated by a dielectric sheet. Line Impedance and attenuation constants, resulting from a specific geometry, are particularly relevant to the application engineer. Back in the time, engineers faced the challenge of little computation power and limited accuracy in measurement technology. In order to build a sufficiently accurate theory, an analytical approach was necessary. However, today's engineers have a large amount of computational power at their disposal. A numerical simulation which can handle complex shapes and an inhomogeneous dielectric filling much easier may offer better approximations to reality. Comparing these approaches shows limitations of the analytic theory and builds trust towards simulating planar waveguides. | MH |

GLOSSARY

α_c	ohmic attenuation in dB/m
α_d	dielectric attenuation in dB/m
k	relative dieelectric constant ε_r of the dielectric sheet, seperating the two metal strips
k_a	$(k+1)/2$ average of dielectric constants of sheet and free space [8]
k_{FR4}	dielectric constant of FR-4 material
k_{PTFE}	dielectric constant of Teflon material
ε_r	equal to relative dieelectric constant k
w	width of the conductor
w'	$w^{'} = w + \Delta w$ - edge correction
t	thickness of the conductor
h	height of the dielectric sheet
Z_0	line impedance
e	Naperian base
λ	wavelength
λ_0	wavelength in free space
c_0	light velocity in free space
R_c	$= 377\Omega$ - wave resistance in free space
μ	magnetic permeability of the metal strips
ρ	electrical resistivity of the metal strips
f	frequency in Hz
R_s	effective skin resistance of the metal strips
L	inductance
q	dielectric filling fraction

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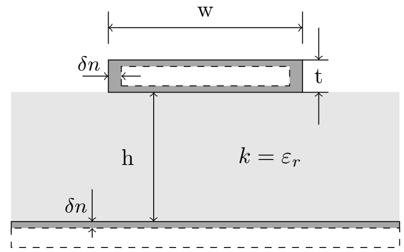

Fig. 1. Illustration of the example geometry with specified parameters according to the listed notation.

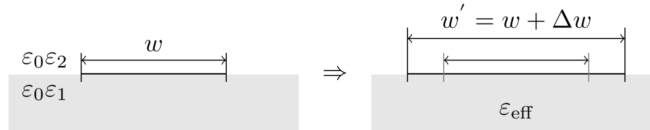

Fig. 2. Transformation of the geometry and applying an effective dielectric constant to calculate line impedance Z_0. Note the assumption $t = 0$ to calculate line impedance.

II. THEORY

Analysing losses in planar waveguide structures similar to the presented structure in Fig. 1, imposes various challenges. The geometry divides into two half spaces introducing an inhomogeneous dielectric filling as is shown in Fig. 1. This complicates the analysis due to continuity conditions of the electromagnetic fields. In [7], Wheeler proposes a solution, using a conformal mapping approach to transform the geometry to only one dielectric filling. Using this conformal mapping and by applying an effective dielectric constant to the transformed dielectric sheet, Wheeler [8] was able to calculate line impedance based on geometry information. The geometry is hereby altered to a slightly different shape to consider fringe fields at the edges (edge correction). However, this technique is based on some simplifications such as an approximate formula for the conformal mapping or conductor thickness being set to zero. Figure 2 illustrates this procedure. The original formula [8] yields only little accuracy for narrow strips but is sufficiently accurate for wide strips.

$$w/h \geq 2: \qquad w' = w + \Delta w \text{ , with } \Delta w = \frac{t}{\pi}\ln\left(\frac{2h}{t}+1\right)$$

$$\alpha_c = \frac{R_s}{Z_0 h}\cdot\frac{8.68}{\left(\frac{w'}{h}+\frac{2}{\pi}\ln\left[2\pi e\left(\frac{w'}{2h}+0.94\right)\right]\right)^2}\left[\frac{w'}{h}-\frac{w'/(\pi h)}{w'/(2h)+0.94}\right]\left(1+\frac{h}{w'}+\frac{h}{\pi w'}\cdot\left[\ln\left(\frac{2h}{t}+1\right)-\frac{1+\frac{t}{h}}{1+\frac{t}{2h}}\right]\right) \tag{1}$$

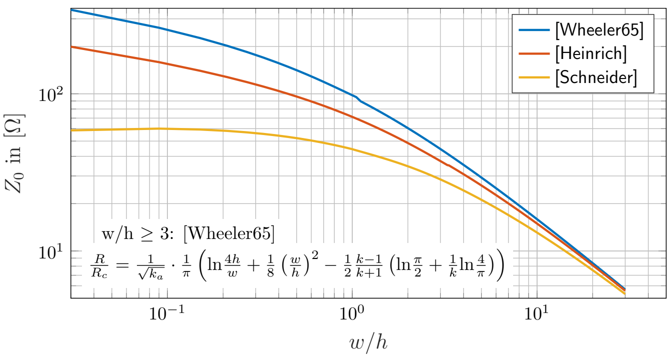

Fig. 3. Comparison of line impedance calculation by various authors

For that reason, it underwent some empirical corrections over a decade later, made possible by increased calculation power and more accurate measurement data [9]. Such a corrected formula is given by Heinrich [3] in (2) and (3) for different shape ratios w/h (compare Fig. 3).

$$w/h \leq 3.3: \qquad Z_0 = \frac{120\Omega}{\sqrt{2(k+1)}}\cdot \left(\ln\left[\frac{4h}{w}+\sqrt{16\left(\frac{h}{w}\right)^2+2}\right]-\frac{1}{2}\cdot\frac{k-1}{k+1}\left[0.452+\frac{0.242}{k}\right]\right) \tag{2}$$

$$w/h \geq 3.3: \qquad Z_0 = \frac{188.5\Omega}{\sqrt{k}}\cdot \left(0.44+\frac{w}{2h}+\frac{k+1}{2\pi k}\left[1.452+\ln\left(\frac{w}{2h}+0.94\right)\right]+0.082\frac{k-1}{k^2}\right)^{-1} \tag{3}$$

Having calculated line impedance makes a prediction of ohmic losses feasible. Pucel et al. [5] use Wheeler's incremental inductance rule (4) [8],[7] to estimate ohmic attenuation.

$$R = \frac{1}{\mu_0}\sum_j R_{sj}\frac{\partial L}{\partial n_j} \tag{4}$$

Using effective skin resistance to model the conductor resistivity (5) leaves the challenge to determine the conductor inductance L (6)

$$R_{sj} = (\pi f \mu \rho_s)^{-\frac{1}{2}} \tag{5}$$

$$L = \sqrt{\mu_0\epsilon_0}Z_0\cdot f\left(\frac{w'}{h},\frac{t}{h},k\right) \tag{6}$$

Giving accurate expressions for the n deviations of the inductance, using Wheeler's conformal mapping approach, including conductor thickness, requires lengthy, yet straight forward calculations that result into (1), given by Pucel et al. [5]. It gives the conductor attenuation α_d in [dB/m] for shape ratios $w/h \geq 2$. Pucel et al. give formulas for all shape ratios in their publication which is not reproduced at this point, due to space limitations.

Finally an equation for dielectric attenuation α_d in [dB/m] is given by Schneider [6] with the equations (7)-(9). The superior accuracy of this empirically found formula given in (9) has been praised in many subsequent publications, such as [9]. Using the effective dielectric constant (7)

$$\epsilon_{\text{eff}} = \frac{\epsilon_r+1}{2}+\frac{\epsilon_r-1}{2}\left(1+\frac{10h}{w}\right)^{-\frac{1}{2}} \tag{7}$$

and the true dielectric filling fraction (8),

$$q = \frac{\frac{1}{\epsilon_{\text{eff}}}-1}{\frac{1}{\epsilon_{\text{r}}}-1}\cdot\tan\delta \tag{8}$$

the dielectric losses (9) α_d can be calculated.

$$\alpha_d = \frac{20\pi}{\ln 10}\cdot q\cdot\frac{1}{\lambda} \qquad \text{using } \lambda = \frac{\lambda_0}{\sqrt{\epsilon_{\text{eff}}}} \tag{9}$$

The attenuation constants α_d and α_c are given in [dB/m] and require no additional post-processing. | MH |

III. SIMULATION

CST MICROWAVE STUDIO® [1] and its time domain solver is used to evaluate the losses in a microstrip model. Line impedance, dielectric losses, and ohmic losses are determined for different geometry and material properties.

The used CAD-model, which is shown in Fig. 4, consists of a rectangularly shaped microstrip, that is separated from the ground-plane by dielectric material. The ground-plane is modeled using a rectangular shape of the same length and width like the dielectric material. The isolators tangent-loss and permittivity are assumed to be constant in the simulated frequency range. Multiple permittivity values are used, the tangent-loss is chosen to be 10^{-4}. Lossy metal with different conductivity values is used for metallization.

For utilization of the model symmetry a magnetic symmetry boundary is used. The ground plane is simulated as electric boundary. The excitation is performed by one waveguide port. The free space boundaries are chosen to be simulated as open boundaries. Few mesh elements are added between geometry and open boundaries. Figure 4 gives an overview of the used boundary conditions.

For reduction of simulation errors resulting from the edges on the microstrip geometry, edge refinement is applied. | RS |

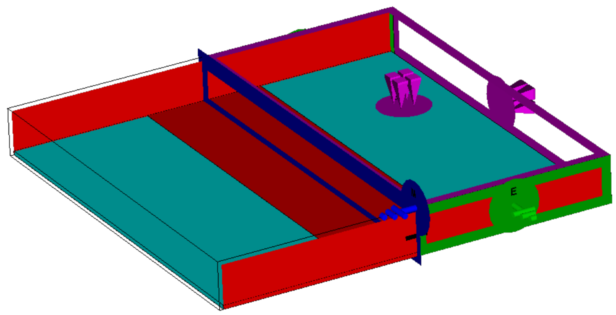

Fig. 4. CST MICROWAVE STUDIO® [1] - simulation assembly and boundary conditions. Colors: green - electrical boundary, purple - open boundary, blue - magnetic symmetry plane

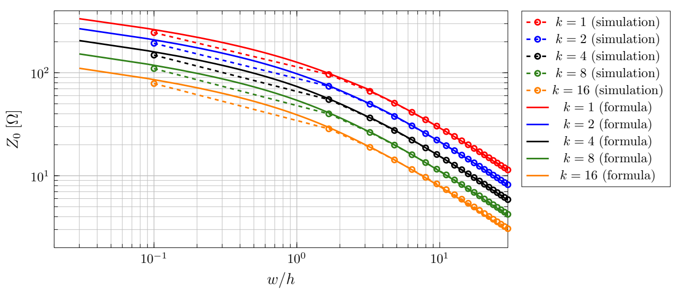

Fig. 5. Line impedance for different k - Heinrich [3] formula in comparison to simulation results

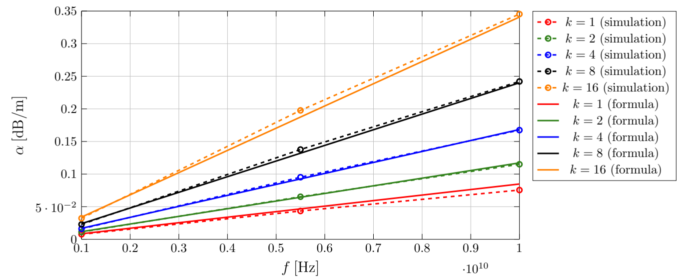

Fig. 6. Dielectric attenuation for different k - Schneider [6] formula in comparison to simulation results

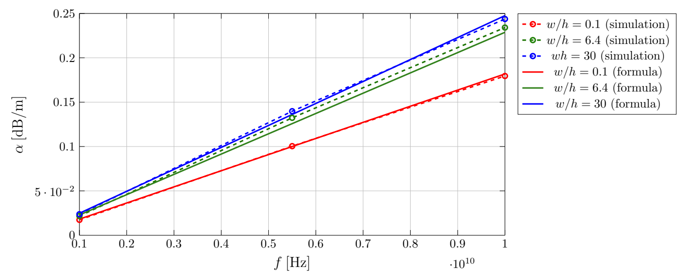

Fig. 7. Dielectric attenuation for different w/h - Schneider [6] formula in comparison to simulation results

IV. RESULTS

The shown diagrams compare the simulation results to the related approximate formulas. Simulation results are expressed as dashed lines with circles for actually simulated values whereas the approximate formulas are expressed with solid lines.

A. Line Impedance

The simulated line impedance is compared to estimations from Wheeler [8], Schneider [6] and Heinrich [3]. Simulations are performed for different values of w/h as well as different values of k. For $w/h \gg 10$ all three formulas converge to the simulation results. For small values of w/h Heinrich's estimation seems to be best, as it is very close to the simulation results as shown in Fig. 5), whereas Schneider's estimation is too low and Wheeler's estimation is too high.

Heinrich's estimations are more recent than Wheeler's and Schneider's estimations. Earlier formulas were mainly based on theoretical simplifications (chapter II) with high errors for uncommonly used geometry properties, whereas recent formulas are fitted using simulation and measurement data. Therefore it can be assumed, that the simulation, as well as Heinrich's formula, deliver the most accurate results for the line impedance.

B. Dielectric Losses

The simulated dielectric losses are compared to estimations from Schneider [6]. Different k are simulated. Specially for commonly used k ($k_{\text{PTFE}} \approx 2$ and $k_{\text{FR4}} \approx 4$) Schneider's estimations show very good agreement to the simulations results as shown in Fig. 6. In general Schneider's formula is slightly lower than the simulation results for $k > 8$ and slightly higher for $k = 1$, which might be neglectable.

Comparing Schneider's estimation [6] to simulation results reveals, that best agreement is achieved especially for high ($w/h = 30$) and low values ($w/h = 0.1$) of w/h as shown in Fig. 7.

Overall Schneider's formula shows high agreement with the simulation results for common k and high as well as low w/h. This leads to the assumption that Schneider's formula is especially developed for commonly used k as well as very narrow and wide microstrips. As the disagreement is small both methods seem to be equally appropriate for dielectric loss evaluation.

C. Ohmic Losses

The simulated ohmic losses are compared to formulas by Pucel et al. [5]. Different ρ, w/h and k are used for the simulations.

The disagreement between simulated and estimated ohmic losses for different k increases when using higher values. For high values of k the frequency dependent disagreement is not neglectable as shown in Fig. 8. Nevertheless, the general qualitative behavior is similar in simulation and formula.

Smaller disagreement occurs when using different values of ρ. The disagreement increases with higher values of ρ as shown in Fig. 9. The influence of ρ on the disagreement between simulation and formula is small in comparison to the influence of k.

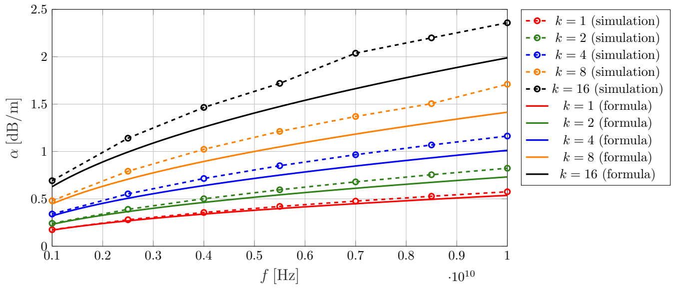

Fig. 8. Ohmic attenuation for different k - Pucel [5] formula in comparison to simulation results

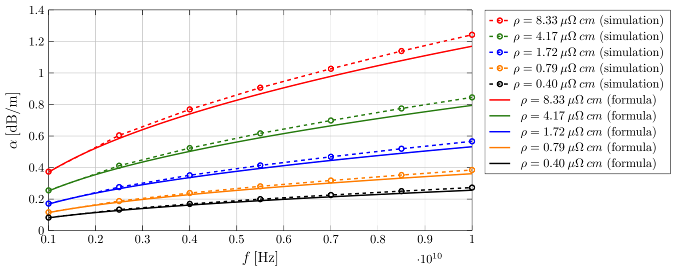

Fig. 9. Ohmic attenuation for different ρ - Pucel [5] formula in comparison to simulation results

Also different w/h are simulated. Especially for small values of w/h increased disagreement occurs as shown in Fig.10.

Overall the ohmic losses show the same qualitative behavior comparing simulations and formula. However, a not neglectable frequency dependent disagreement occurs. The formula is consistently lower than the simulation results. The same behavior has also been shown through measurement conducted, by Pucel [5]. As Pucel et al. [5] use Wheeler's conformal mapping [7], ohmic losses are wrongly predicted. Through measurement conducted, by Pucel [5], it is known that theory consistently predicts lower ohmic losses. The numerical simulation yields larger ohmic losses, so a better accuracy can be assumed. This is why the simulation might deliver a more accurate ohmic loss prediction. | RS |

V. SUMMARY

The theoretical analysis shows good agreement with the numerical solutions. Especially dielectric losses can be predicted very accurately according to the simulation. Line impedance calculation, despite optimized formulas, still shows small tolerances, at delivering close approximations for very narrow strips. This is due to approximate formulas for the conformal mapping and the incorrect assumption of considering transversal electromagnetic waves propagating inside the waveguide. These aspects introduce an error to calculate ohmic losses accurately. The electrical properties resulting from the geometry can be calculated, using numerical simulation when edges are properly treated. CST MICROWAVE STUDIO® uses an analytical model to help the numerical analysis in avoiding incorrectly large values for the electromagnetic fields around the edges. Using a mesh grid refinement around the shape edges makes the simulation more accurate. | MH |

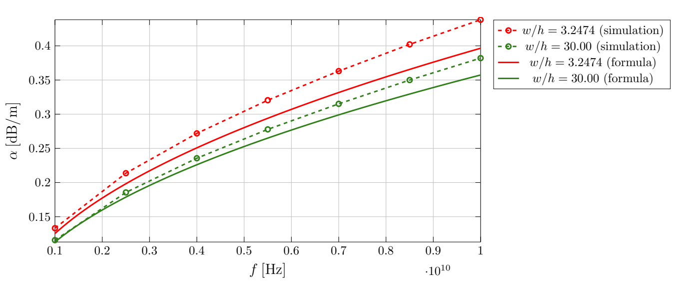

Fig. 10. Ohmic attenuation for different w/h - Pucel [5] formula in comparison to simulation results

VI. OUTLOOK

Computer-aided design can also be used to fit and find accurate formulas to predict losses in microstrips. Hammerstad et al. [2] used this approach to find accurate formulas for attenuation constants derived from Q factor estimation. However for numerical simulation this approach holds challenges to calculate correct eigenmodes in resonant problems while considering losses. Kirschning et al. [4] used an empirical approach to find an accurate, frequency dependent formula to determine the effective dielectric constant which is crucial to predict dielectric losses. Over a very large frequency range, the linear, empirical approximation proposed by Schneider [6] might not be sufficient. | MH |

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